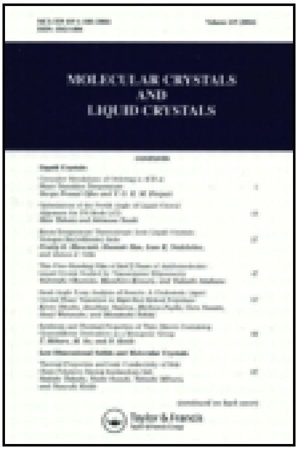

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Field Induced SmC$_\alpha$*-SmC Transition and Discrete Soliton Excitation

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The helical structure of the chiral smectic C$_\alpha$ phase (SmC$_\alpha$*) is unwound by an electric field and a field induced SmC$_\alpha$*-smectic C phase transition occurs at a certain field strength, which is similar to the chiral smectic C phase (SmC*) where the transition is interpreted as a condensation of solitons. The pitch of SmC$_\alpha$* without the field is very short and the soliton excited at the phase transition should be a discrete type, in contrast with SmC* where the pitch is large enough and a continuum theory is applied. It is shown that the SmC$_\alpha$*-SmC transition is of the second order like the SmC*-SmC transition, while the interaction between discrete solitons is quite short range and the condensation process of solitons occurs drastically. The dependence of wave number of the helical structure on the field strength is possibly something like staircase. At high field region, discrete soliton lattices of short period become unstable showing fragile property.

Keywords: discrete soliton; interaction between solitons; SmC*-SmC transition; SmC$_\alpha$*-SmC transition; soliton excitation

1. INTRODUCTION

In some antiferroelectric smectic materials, a chiral smectic C$_\alpha$ phase (SmC$_\alpha$*) appears in the low temperature side of smectic A phase [1]. The phase SmC$_\alpha$* is considered to be ferroelectric and has a helical structure [2], which is similar to chiral smectic C phase (SmC*). Both phases are unwound by an electric field, and at a certain field strength a phase transition to smectic C phase (SmC) occurs [1,3–6]. In SmC* the pitch without field is sufficiently large so that the phase is described in the framework of continuum theory. The transition between SmC* and

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SmC is interpreted as a soliton condensation process, in which the soliton is a solution to the static sine-Gordon equation [7,8]. On the other hand, in $\mathrm{SmC}_\alpha{}^*$ the pitch is very short, that is 4 or 5 layers [2]. Accordingly, it is not appropriate to use the continuum theory. In the present study, the $\mathrm{SmC}_\alpha{}^*$-SmC transition is analyzed on a standpoint of discrete soliton excitation, in which we focus upon how the transition properties are changed by the discrete description of sine-Gordon soliton, because the nature of discrete solitons is not clear so far.

A free energy of Ginzburg-Landau type in the continuum theory for SmC* [9] is generalized to a discrete form to be applicable to $\mathrm{SmC}_\alpha{}^*$. Only the case of constant tilt angle is considered in the present study, and the minimum condition of free energy is solved numerically. It is shown that the transition is continuous showing the interaction between discrete solitons to be repulsive. However, the interaction range is quite short and consequently the soliton condensation occurs drastically at a field just below the critical field, which is compared with the behaviour at the SmC*-SmC transition. The change of wave number of the helical structure for increasing field strength is suggested not to be continuous but something like a staircase.

2. SOLITON CONDENSATION AT THE SMC*-SMC TRANSITION

In the first place, we explain a mechanism of the SmC*-SmC transition as the soliton condensation process in SmC [8]. The free energy in the field is given by [9]

$$F = \int \left[\frac{1}{2} a\theta^2 + \frac{1}{4} b\theta^4 + \frac{1}{2} K \left(\frac{d\theta}{dz} \right)^2 + \frac{1}{2} K\theta^2 \left(\frac{d\varphi}{dz} - q_0 \right)^2 - E\theta \cos\varphi \right] dz, \tag{1}$$

in which θ denotes a tilt angle, φ an azimuthal angle of tilt direction and q_0 the wave number at vanishing field. Under the condition of constant tilt ($\theta = \theta_0$) which applies to a low temperature region of SmC* ($|a|$ is sufficiently large), the Euler-Lagrange equation is given by

$$\frac{d^2\varphi}{dz^2} = \frac{E}{K\theta_0} \sin\varphi. \tag{2}$$

A periodic solution to Eq. (2) is derived as

$$\varphi = \sin^{-1} \mathrm{sn} \left(\sqrt{\frac{E}{K\theta_0}} \frac{z - z_0}{\kappa} ; \kappa \right), \tag{3}$$

where sn(x, κ) denotes Jacobi's sn-function. From the condition of minimum free energy density, the integral constant κ in Eq. (3) is determined by the following equation,

$$\frac{\kappa}{E(\kappa)} = \frac{4}{\pi}\sqrt{\frac{E}{K\theta_0}}, \tag{4}$$

where $E(\kappa)$ is the complete elliptic integral of second kind. The periodic solution (3) shows SmC*.

We have also a soliton solution to Eq. (2),

$$\varphi = 4\tan^{-1}\tanh\left(\sqrt{\frac{E}{K\theta_0}}\frac{z-z_0}{2}\right) + \pi, \tag{5}$$

and an excitation energy of the soliton at SmC, ΔF, is derived by substituting Eq. (5) into Eq. (1) as

$$\Delta F = 4\sqrt{\pi}K\theta_0^2 q_0\left(\sqrt{\frac{E}{E_c}} - 1\right), \tag{6}$$

in which the critical field is given by $E_c = (\pi/4)^2 K\theta_0 q_0^2$. The excitation energy of soliton vanishes at the critical field, and for the field smaller than that, a nucleation of soliton occurs. However, due to a repulsive interaction between solitons, soliton density remains finite, where solitons take an arrangement with equal separation forming the periodic structure described by Eq. (3). Because of this, the solution (3) is called a soliton lattice. The soliton density is equivalent to the wave number q, which is calculated as

$$\frac{q}{q_0} = \frac{1}{2\kappa K(\kappa)}\sqrt{\frac{E}{K\theta_0}}, \tag{7}$$

where $K(\kappa)$ is the complete elliptic integral of first kind.

In case the change of the tilt $\theta(z)$ is taken into account which applies to a high temperature region in SmC* ($|a|$ is small), various phenomena occur. Especially, the interaction between solitons turns to attractive, and consequently the SmC*-SmC transition becomes first order, in which the soliton lattice with finite q transforms to uniform SmC discontinuously [7,8]. Though these phenomena are very interesting, details are skipped here since we restrict the present discussion on SmC$_\alpha$*-SmC transition to the case of constant tilt.

3. SMC$_\alpha$*-SMC TRANSITION

3.1. Discrete Form of Free Energy for SmC$_\alpha$*

Generalization of the free energy (1) to the discrete form is straightforward. The tilt angle and the azimuthal one for the mean of orientations of molecular long axes in i-th layer are denoted by θ_i and φ_i, respectively. By taking into account the phase variables, θ_i and φ_i, Eq. (1) is generalized for SmC$_\alpha$* as

$$F = \sum_i \left[\frac{1}{2} a\theta_i^2 + \frac{1}{4} b\theta_i^4 - K\cos(\theta_{i+1} - \theta_i) \right. \\ \left. - K\theta_{i+1}\theta_i \cos(\varphi_{i+1} - \varphi_1 - \delta) - E\theta_i \cos\varphi_1 \right]. \tag{8}$$

It is noticed that the same characters are utilized for the electric field and the coefficients though each corresponding characters in Eqs. (1) and (8) are different, at least in dimension. Under the condition of constant tilt ($\theta_i = \theta 0$), the minimum condition of F is given by

$$\sin(\varphi_{i+1} - \varphi_i - \delta) - \sin(\varphi_1 - \varphi_{i-1} - \delta) - e\sin\varphi_i = 0, \tag{9}$$

where $e = E/K\theta_0$. Equation (9) is a discretized form of the sine-Gordon equation (2). Periodic solutions to Eq. (9) are obtained numerically, with $\varphi_{i\pm p} = \varphi_i + 2\pi m$, ($i$ = 2,3,..., p, m = 1, 2, ...), and the wave number q is equal to m/p.

3.2. Numerical Results

We take $\delta = \pi/2$, which corresponds to the case where a phase with period 4 is stable in the absence of the field. As to the wave numbers of periodic solutions practically taken into account, first we choose $1/n$, (n = 2, 3,...,10) and next pick up some of numbers from Farey series, 4/17, 3/13, 2/9, 3/14, 4/19, 2/11, and 2/13. In Figure 1, profiles of period 7 solutions are shown for several field strengths, where e_c is the critical field at which the transition to SmC occurs. It is noticed here that the profile at e_c in Figure 1 is quite similar to a profile of soliton, the reason of which will be explained in the discussion of Figure 3 below. The wave number dependence of free energy per particle, ΔF, is shown in Figure 2, where circles, squares, diamonds and triangles are those at several field values for the wave numbers $1/n$, (n = 2, 3, ..., 10). Solid curves are derived from Eq. (1) together with Eqs. (3) and (7) based on the continuum theory. The dependence is similar to the continuous one qualitatively, and within the accuracy

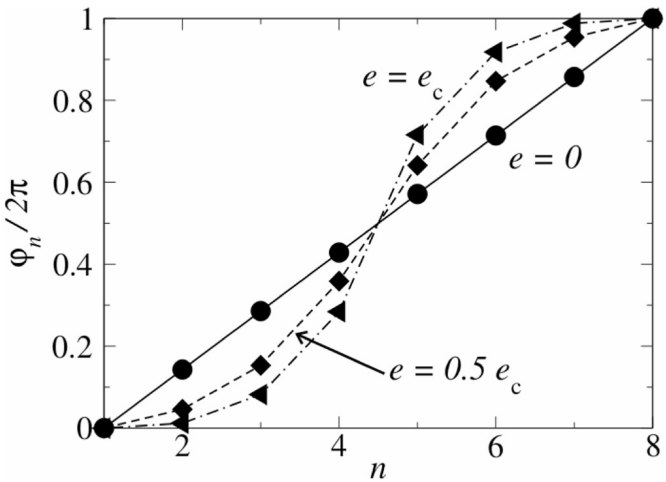

FIGURE 1 Profile of periodic solution of period 7.

of the present numerical analysis (of the order 10^{-8}), the SmC$_\alpha$*-SmC transition is concluded to be continuous. In practice, we also find a similarity in the behaviours of the soliton tails which have an intimate relation to the order of transition. The soliton tail in the continuum theory has the form of a simple exponential decay for the continuous phase transition in contrast with the oscillatory decay for the

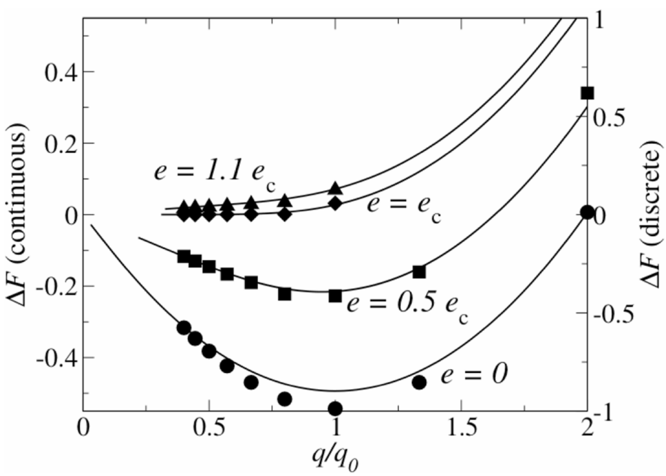

FIGURE 2 Free energy as a function of wave number q.

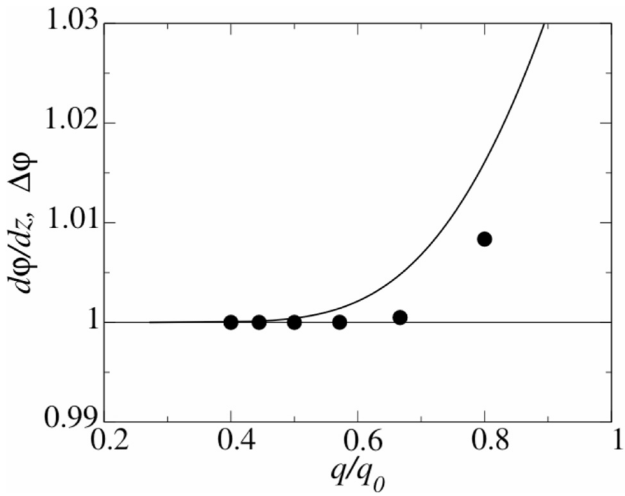

FIGURE 3 q-dependence of the difference $\Delta\varphi$ at the centre of soliton.

discontinuous transition [7]. Here, we assume a form for the discrete soliton as

$$\varphi_i = 2\pi + \varepsilon e^{ki}. \tag{10}$$

Then, by a linear analysis of Eq. (9) together with form (10), we obtain the positive value for k as

$$k = 2\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{e}{\cos\delta}}\right). \tag{11}$$

Thus, the simple exponential decay form of the soliton tail is shown, as the continuous soliton's. It is noted that we have no solution with $q = 1/2$ and $1/3$ at $e/e_c = 1.0$ and 1.1, and also no solution with $q = 1/4$ for a field higher than these, which is compared with the continuous scheme where the periodic solution with any wave number exists as in Eq. (3). This instability of periodic solutions with large wave number at high field shows the fragile characteristics of the discrete soliton lattice.

A linear dependence near the origin ($q \approx 0$) is commonly observed at both discrete and continuous soliton lattices in Figure 2, which means that the interaction between solitons is negligible for soliton density in that range. Numerically, ΔF is of the order 10^{-5} in the unit $k\theta_0^2$ for q from 1/10 to 1/7 and of the order 10^{-4} for $q = 1/6$ and 1/5 at $e = e_c$. However, we see a remarkably broad linear region for discrete soliton lattice in comparison with that for the continuous one. This fact indicates that the range of interaction between discrete solitons is shorter than that between continuous solitons. Being the profile of soliton is deformed at soliton lattice due to the interaction, the short range property is also observed at a shape change of soliton lattice for various wave numbers, which is characterized by a difference $\Delta\varphi$ $(= \varphi_{i+1}\ \varphi_i)$ at the centre of soliton in the soliton lattice, as shown in Figure 3 with arbitrary unit for the ordinate. In the figure, $\Delta\varphi$ is plotted for $q = 1/10, 1/9, \ldots,$ and $1/5$ together with a solid curve showing the continuum correspondence, i.e., the maximum of $\mathrm{d}\varphi/\mathrm{d}z$ derived from Eqs. (3) and (7). We see that any change of $\Delta\varphi$ is not realized for q from $1/10$ to $1/7$ and starts at $1/6$ corresponding to a change in ΔF in Figure 2, which is compared with the change of $\mathrm{d}\varphi/\mathrm{d}z$ observed to start at $q/q_0 \approx 0.45$. Now, regarding the profile of soliton, the insensitivity of soliton lattice profile to the wave number of small value together with the linear dependence of ΔF on the wave number gives a reason of the notion below Figure 1 that the profile for $q = 1/7$ at e_c in Figure 1 is assumed to resemble the soliton profile, although we have not obtained a soliton solution numerically.

Finally we show an electric field dependence of wave number in Figure 4, where a stable range for each phase characterized by the wave number indicated is depicted within the restriction of 16 wave numbers as mentioned in the head of this section. The solid curve for the continuous case is derived from Eq. (7) under the equilibrium condition given by Eq. (4). Corresponding to the shortness of the

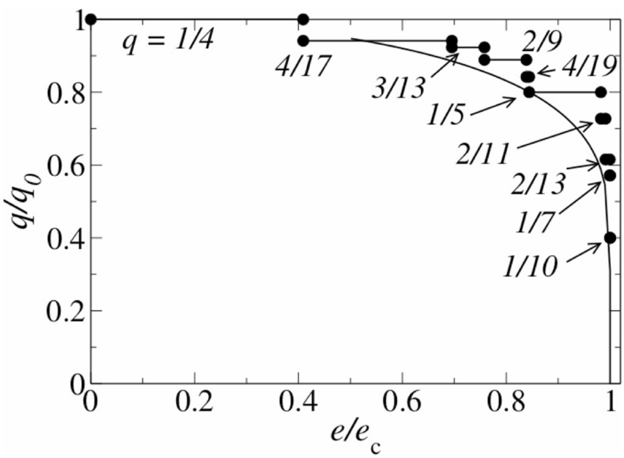

FIGURE 4 Field dependence of the wave number.

interaction range, a drastic condensation process just below the critical field is observed for the discrete soliton. Wide stable regions for certain phases, especially the one with wave numbers 1/5, are also remarkable, suggesting a staircase for the *q*-*e* relation. Practically, some phases, *e.g.*, with wave numbers 1/6 and 3/14, do not appear as a thermal equilibrium phase. If the *q*-*e* relation is continuous like the case of continuous system, any phase should appear even in a short interval of *e*. In this respect, the *q*-*e* relation is suggested to be not continuous, at least in the range of large wave number, or low field. In the present stage, it is not clear whether the relation gives a harmless staircase or not.

4. SUMMARY

The field induced SmC_α^*-SmC phase transition is studied on a standpoint of discrete soliton condensation, which is compared with the field induced SmC^*-SmC transition whose mechanism has been interpreted as a condensation of sine-Gordon soliton. Within the accuracy of the present numerical analysis, the SmC_α^*-SmC transition is concluded to be continuous like the case of SmC^*-SmC transition under the condition of constant tilt. The effect of discreteness of the system appears in the shortness of interaction range between solitons in comparison with the one acting between continuous solitons, which leads to the sharp increase in the wave number of soliton lattice just below the critical field at the SmC_α^*-SmC transition. The field dependence of wave number does not seem to be continuous but makes a staircase, at least for large wave number. The instability of soliton lattice with large wave number is observed at high field, which suggests some fragility of the discrete soliton lattice.

Whether the electric field dependence of wave number makes a devil's staircase or a harmless one is an interesting question. It is also a crucial problem how the range of staircase extends. In case the staircase ends at a finite wave number and remains only in the region of rather large wave number, there is no discrepancy. However, if the staircase continues to infinitely small wave number, we have to reexamine the continuity of phase transition at the critical field. Experimentally, something like transitional behaviours within SmC_α^* have been observed at an apparent tilt angle [1] and at birefringence together with apparent tilt [4], and a change of apparent tilt angle observed with increasing field strength at an early stage of investigation of SmC_α^* has been interpreted as a staircase [10]. Orihara *et al.* have well reproduced the birefringence data on the basis of phenomenological free energy with second neighbour interlayer interaction [4]. In this context, the investigation of the field dependence of wave number will give a clue for clarifying the mechanism of behaviours observed in the SmC$_\alpha^*$ phase, even though the present study is restricted to the condition of constant tilt. The variation of tilt angle leads to a large variety of interesting behaviours as expected from the analogous SmC*. However, it is necessary to obtain concrete results under the restriction of constant tilt in order to understand effect of discreteness of the system.

REFERENCES

[1] Hiraoka, H., Takanishi, Y., Takezoe, H., Fukuda, A., Isozaki, T., Suzuki, Y., & Kawamura, I. (1992). *Jpn. J. Appl. Phys.*, *31*, 3394.
[2] Mach, P., Pindak, R., Levelut, A.-M., Barois, P., Nguyen, H. T., Huang, C. C., & Furenlid, L. (1998). *Phys. Rev. Lett.*, *81*, 1015.
[3] Bourny, V. & Orihara, H. (2001). *Phys. Rev. E 63*, 021703.
[4] Orihara, H., Naruse, Y., Yagyu, M., Fajar, A., & Uto, S. (2005). *Phys. Rev. E 72*, 040701.
[5] Meyer, R. B., Liebert, L., Strzelecki, L., & Keller, P. (1975). *J. Phys. Lett. France*, *36*, L-69.
[6] Meyer, R. B. (1977). *Mol. Cryst. Liq. Cryst.*, *40*, 33.
[7] Yamashita, M. (1985). *Prog. Theor. Phys.*, *74*, 622.
[8] Yamashita, M. (1991). *Solitons in Liquid Crystals*, Lam, L. & Prost, J. (Eds.), Springer-Verlag: Berlin, Chap. 10.
[9] Pikin, S. A. & Indenbom, V. L. (1978). *Sov. Phys. -Usp.*, *21*, 487.
[10] Hiraoka, H., Takanishi, Y., Skarp, K., Takezoe, H., & Fukuda, A. (1991). *Jpn. J. Appl. Phys.*, *30*, L1819.